

# Developing a Robust Process for Design Margin Calculation Through Modal Scaling and Harmonic Analysis of Engine Mounted After Treatment System

Zubair Shaikh, Sudarshan Gajre, Kishor Deshmukh, Jotiprasad Shete



**Abstract-** The last few years have seen an increase in the complexity of emission standards. This has caused OEMs to start using Exhaust Gas Processors (EGP) or Exhaust After Treatment Systems (EATS) in complex integration. These systems are usually mounted on vehicle chassis or engine body with the help of mounting straps or brackets. The arrangement of the system leads to road loads and/or engine vibrations being transferred and causing damage to its components. New BS VI engines need some AT (After treatment) components to be mounted directly on the engine and others to the chassis. The components that are directly mounted on engine are called close couple AT system which are subjected to engine harmonic vibrations which falls under forced vibration criteria. Most of the AT system are subjected to random vibrations and industry has a well-defined procedure to address the problem, but for harmonic vibrations there is no specific approach. In order to optimize the structural durability of close couple AT system against HCF (high cycle fatigue) fatigue harmonic vibration, there is a need for well-defined test analysis correlation procedure. The objective of this work is defining and documenting a robust process for design margin through calculation modal scaling and harmonic analysis of close couple AT system through test analysis correlation activity. Analytical, numerical and testing data were compared and conclusions are drawn. More case studies can be added to this work in order to validate the test analysis correlation activity and boost the degree

of confidence. Future study may include recording additional failure modes in addition to the harmonic HCF failure mode.

**Index Terms:** After Treatment System (ATS), High Cycle Fatigue (HCF), Test Analysis Correlation, Structural Durability, Harmonic Vibration, forced vibration, Close Couple (CC) ATS, (EGP) Exhaust Gas Processor.

## I. INTRODUCTION

The Indian automotive industry has migrated from BS-IV (Bharat stage IV) to BS-VI (Bharat Stage VI) emission norms from 1st April 2020 which demands some complex design and integration of Exhaust After Treatment Systems (EATS). New BS VI engines need some components to be mounted directly to the engine and others to the chassis. Harmonic load due to engine firing orders is dominating load in engine mounted AT systems as compared to road load vibrations in chassis mounted AT system. The architecture of the Engine mounted system is also different than chassis mounted, so typical chassis-mounted system fixtures are also not applicable. There are various failure modes affecting the durability of the engine shown in below Table 1: -

**Table 1: Aftertreatment System Failure Modes**

Sr No.	1	2	3	4	5
Failure Mode	HCF (High Cycle Fatigue) due to Road Vibrations	HCF due to harmonic Load of Engine	Failure due to Rotating Forces	LCF (Low Cycle Fatigue) due to Thermal Expansion and Contraction	Bolt/Clamp and Gasket Failure
Analysis Type	PSD analysis (Power Spectral Density)	Harmonic Analysis/Modal Scaling	Rotating Bending Fatigue Analysis	Thermal Analysis	Fatigue Analysis

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**Figure 1: Aftertreatment System with Close Couple**

Above Figure 1 depicts an AT system with close couple DOC (Diesel oxidation catalyst). There is a requirement for a unique validation technique through simulation and testing because engine mounted AT systems do not work with traditional dynamic analysis and test processes like PSD (power spectral density).



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It is essential to create an analysis methodology to assess the engine mounted AT system's structural durability in order to validate it. One of the crucial elements is also confirming the analysis methodology through test correlation. Engine mount close-coupled subassembly of an aftertreatment system's current analysis methodology is not well defined and validated. To verify the design margin of the engine mounted AT system, it is possible to conduct test-analysis correlation activity using harmonic analysis

## A. Working of After Treatment System

The Aftertreatment system is a series of different components attached one after the other as shown in below figure 2, to reduce the emissions of a vehicle and meet the standards set by the government. It should eliminate NO<sub>x</sub>, CO, HC and PM. There are various units involved in eliminating specific pollutants. Below, the architecture of an Aftertreatment structure is shown.

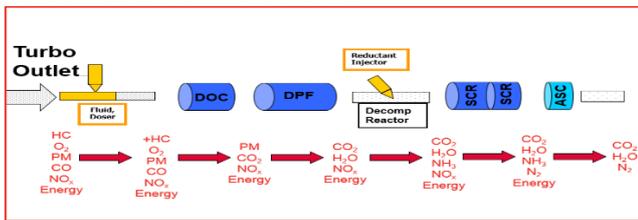


Figure 2:- After Treatment System Working

Clearly, the components involved are:

- Diesel Oxidation Catalyst Chamber
- Diesel Particulate Filter
- Selective Catalytic Reduction Chamber
- Decomposition Reactor
- Urea Dosing System
- Ammonia Slip Catalyst Region

The close couple AT system consists of this extra DOC which is mounted to engine manifold through turbo charger hence contains forced vibration from engine in terms of harmonics. These harmonic vibrations were detected in testing and analysis in order to analyse the structural durability of AT system.

## II. HARMONIC ANALYSIS

Dynamic analysis techniques like harmonic analysis are used to determine how a structure will react to static and harmonic (sinusoidal changing) loads. The natural frequencies and mode shapes are computed using modal analysis as part of the Mode-Superposition based Harmonic analysis approach. The mode-superposition solution is then applied, combining various mode forms to produce a solution. A structure's steady-state response to sinusoidal changing loads acting at a certain frequency is determined via harmonic analysis. A variety of user-defined frequencies are used to conduct the solution. No. Harmonic analysis allows nonlinearities, and even if plastic properties are defined, harmonic analysis will disregard them. There are continual stiffness, damping, and mass effects throughout the entire structure.

The harmonic analysis is explained below with the help of case study 1 that how it is related to static and modal scaling with case study 2.

Case study 1 :-

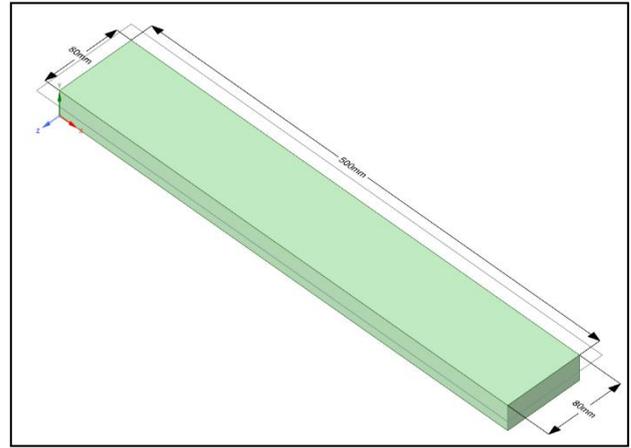


Figure 3: Cantilever Beam

Figure 8 shows a cantilever beam with below dimensions:  
 Length of Beam = 500 mm  
 Width of Beam = 80 mm  
 Thickness of Beam = 20mm  
 Boundary Conditions: -  
 Fixed End: - Slope = 0, Deflection = 0  
 Free End: - Force

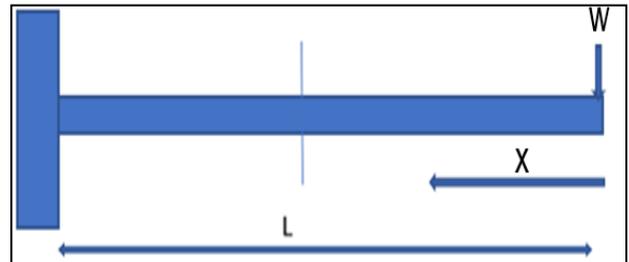


Figure 4: Cantilever Beam Diagram

Figure shows the force applied on free end of cantilever beam.

$$F_0 \sin \omega t = m\ddot{x} + c\dot{x} + kx \text{ ----- (equation of motion)}$$

This equation has solution as

$$X = X_{complimentary} + X_{particular}$$

$$X = \sqrt{\frac{f/k}{(1-r^2)^2 + (2\zeta r)^2}} \text{ ----- (displacement)}$$

$$\phi = \tan^{-1} \left( \frac{2\zeta r}{1-r^2} \right) \text{ ----- (phase angle)}$$

$$M.F. = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \text{ ----- (magnification factor)}$$

$$C_c = 2\sqrt{km} \text{ ----- (Critical Damping)}$$

$$\zeta = \frac{c}{C_c} \text{ ----- (Damping ratio)}$$

$$\omega d = \omega n \sqrt{1 - \zeta^2} \text{ ----- (Damped frequency)}$$

Let,

$$r = \frac{\omega}{\omega_n} \text{ ----- (1)}$$

$$\omega_n = \sqrt{\frac{k}{m}} \text{ ----- (2)}$$

$$k = \frac{3EI}{l^3} \text{ ----- (3)}$$

$$I = \frac{BD^3}{12} = \frac{0.08 \times 0.02^3}{12} = 5.333 \times 10^{-8} \text{ ----- (4)}$$

$$E = 2e11 \text{ N/m}^2 \text{-----(5)}$$

$$L = 0.5 \text{ m} \text{-----(6)}$$

$$k = \frac{3EI}{l^3} = \frac{3 \times 2 \times 10^{11} \times 5.333 \times 10^{-8}}{0.5^3} = 255840 \frac{\text{N}}{\text{m}} \text{-----(from 3,4 and 5)}$$

$$m = \text{density} \times \text{volume}$$

$$= 7850 \frac{\text{kg}}{\text{m}^3} \times (0.5 \times 0.08 \times 0.02) \text{m}^3$$

$$= 6.28 \text{ kg}$$

$$\text{Mass} = 6.28 \times 9.81 = 61.606 \text{ N}$$

Modulus of Elasticity: -

$$E = 2e5 \text{ Mpa} = 2 \times 10^{11} \text{ N/m}^2$$

$\omega$  is the frequency of applied load which is 30 Hz in our case ( 30 Hz =  $30 \times 2\pi = 188.49 \text{ rad/sec}$  )

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{255840}{61.606}} = 64.44 \text{ Hz}$$

$$\omega_n = 64.44 \times 2\pi = 404.904 \text{ rad/sec}$$

$$\text{Therefore, } r = \frac{\omega}{\omega_n} = \frac{188.49}{404.904} = 0.465$$

$$\zeta = \frac{c}{c_c}$$

$$\frac{x}{x_{st}} = \text{M. F. (Magnification Factor)}$$

$$\text{M. F.} = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \text{-----(7)}$$

$$\phi = \tan^{-1} \frac{2\zeta r}{1-r^2} \text{-----(8)}$$

Therefore,

$$\text{M.F} = 1.275 = \frac{x}{x_{st}}$$

$$\text{M. F.} = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \frac{1}{\sqrt{(1-0.465^2)^2 + (2 \times 0.02 \times 0.465)^2}} =$$

$$1.275 \text{ ----- (from 7 \& 8)}$$

$$\phi = \tan^{-1} \left( \frac{2\zeta r}{1-r^2} \right) = \tan^{-1} \left( \frac{2 \times 0.02 \times 0.465}{1-0.465^2} \right) = 1.35^\circ$$

$$X = 3.906 \times 1.275 = 4.98 \text{ mm}$$

Result from Ansys (harmonic analysis): -

**X = 4.8498 mm (Figure)**

Percentage Difference: -

$$\text{Percentage difference} = \frac{4.98 - 4.8498}{4.98} = 2.6\%$$

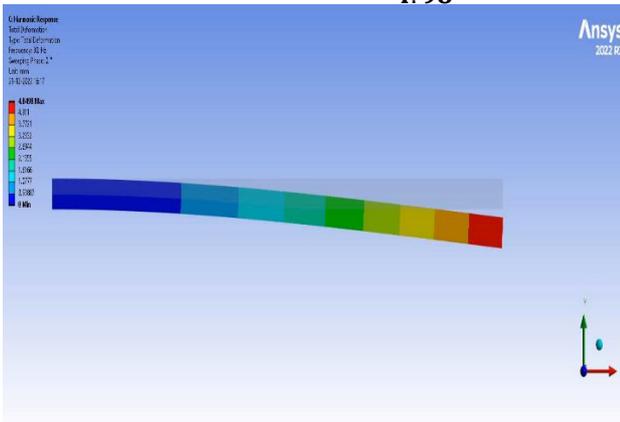


Figure 5: Harmonic Response Cantilever Beam

### III. MODAL SCALING

In order to identify stresses in an item responding in its global modes under a specific boundary and input environment, the modal-scaling technique simply treats normal mode data. For a system to work well while vibrating,

resonance conditions should be avoided because they can result in failure. Modal analysis offers us an understanding of how the design will react to various sorts of dynamic loads at various frequencies, so we may fine-tune the design to minimize resonant vibrations by experimenting with stiffness K, system mass, and damping.

Case study 2: -

For first Natural frequency: - 109.52 Hz

$x \times \omega^2 = \text{Acceleration}$

Response Displacement (x) =  $\text{Acceleration} / \omega^2$

$x = \text{Acceleration} / 2\pi f^2$  therefore,

$x = 7.6527 \times 1052 \pi \times 109.52^2$  Response Displacement: -

$x = 1.616099 \text{ mm}$

Modal displacement D\_Modal = 238.34 mm

Response Displacement D\_Response = 1.61

Scaling Factor (r) =  $D_{\text{Response}} / D_{\text{Modal}} = 1.61 / 238.34 = 6.78e-3$

Therefore,

With the help of scaling factor: -

Modal Stress  $\times$  Scaling factor = Scaled Stress

Scaled stress =  $11979 \times 0.006$

Scaled stress = 81.22 Mpa

From harmonic analysis: - Stress = 83.30 MPa

Percentage difference =  $(83.303 - 81.228) / 81.228 \times 100 = 2.5\%$

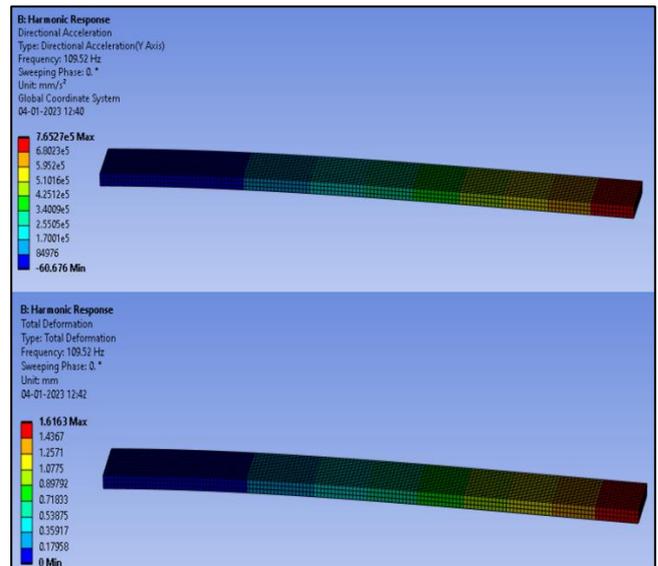
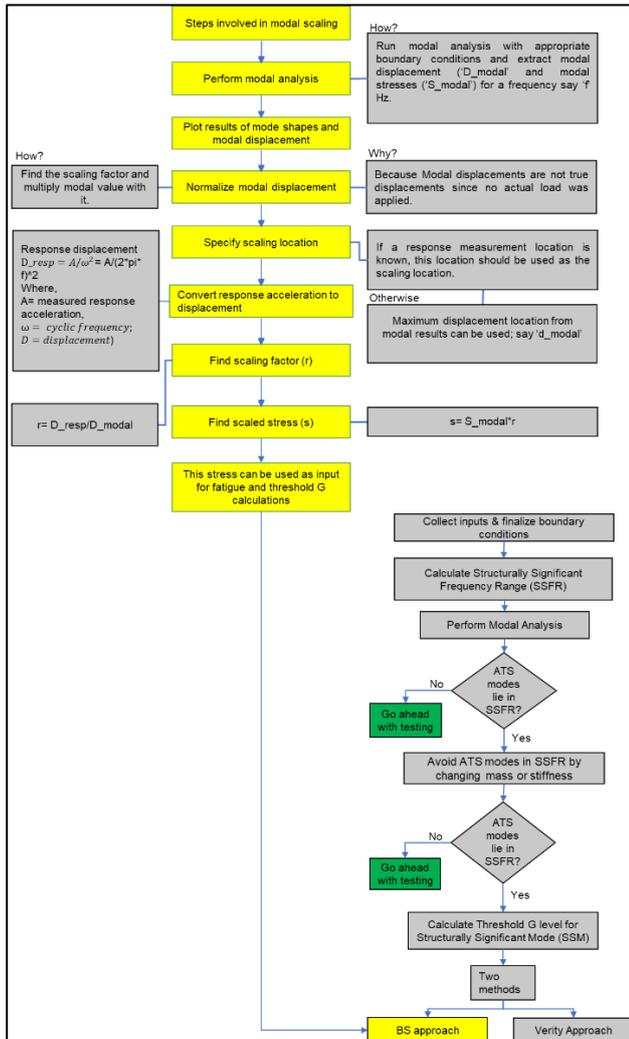


Figure 6: Harmonic Response

Analytical and numerical approach for harmonic/modal scaling is explained with the help of cantilever beam as shown in figure 6 and 2.5 percentage difference was observed in comparison. And the error will make the design conservative and not under design. SSFR: - Structurally significant frequency range. (The frequency range which is more likely to excite resonance.)

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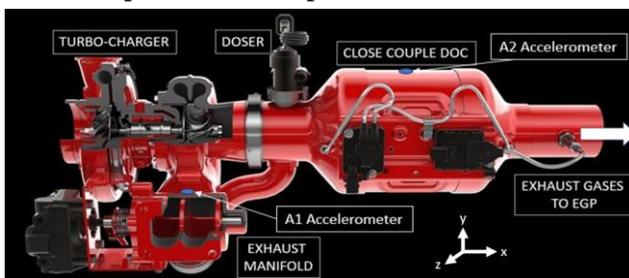


**Figure 7: Modal Scaling Procedure**

Figure 7 in previous page shows the overall procedure of modal scaling process where the displacements found in the modal analysis were normalized using scaling factor and corresponding von mises stress were calculated which were then given as an input to BS approach (British Standard) to find out the design margin with respect to fatigue criteria.

## IV. CASE STUDY: 15-LITER ENGINE WITH CLOSE COUPLE AFTER TREATMENT SYSTEM

### A. Experimental Setup:



**Figure 8: Experimental Setup**

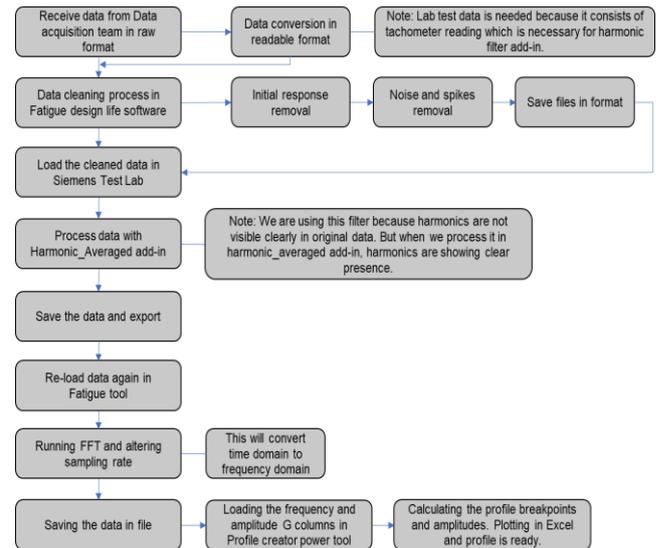
The close couple ATS is mounted on a frame with engine. The engine is run at different speeds and 2 accelerometers A1 and A2 are mounted on the ATS as shown in the above figure 8.

Steady state with full load : (1220rpm-1420 with increment of 10 rpm in each condition.

Speed sweep with full load: 600 – 1900 (Twice)  
 Speed sweep with full load: 1900 – 600 (Twice)  
 Engine Cylinder Displacement: 15 Liter  
 Test coordinate system: X, Y and Z directions of all the test points are the same as the direction of the vehicle coordinate system, the figure is Shown on the right.

Figure shows A1-A2 as the 2 accelerometers. One accelerometer mounted on the close couple for response location vibration measurements and other at the input bracket side for input loading vibration measurements.

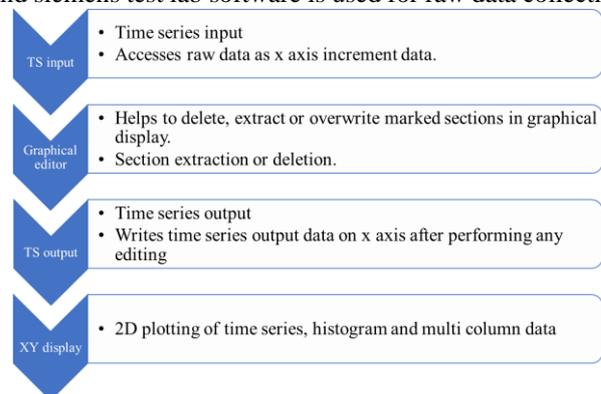
## V. METHODOLOGY FOR HARMONIC PROFILE GENERATION



**Figure 9: Harmonic Profile Generation Process**

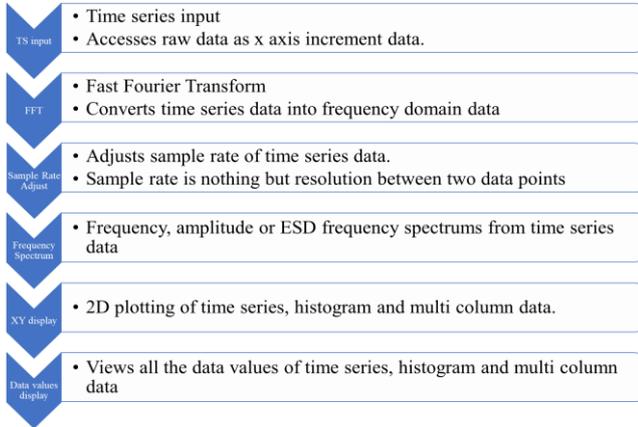
Above figure 9 shows the step-by-step procedure for harmonic profile creation. These profiles were used as input vibration and output response vibration for test analysis correlation activity. Test information are as follows: -

The engine and after treatment system is mounted on a frame structure, and AT system is connected to engine cylinder block with exhaust manifold with replication of on field scenario. Then the engine is run at different conditions to note down the vibrations with the help of accelerometers and siemens test lab software is used for raw data collections.



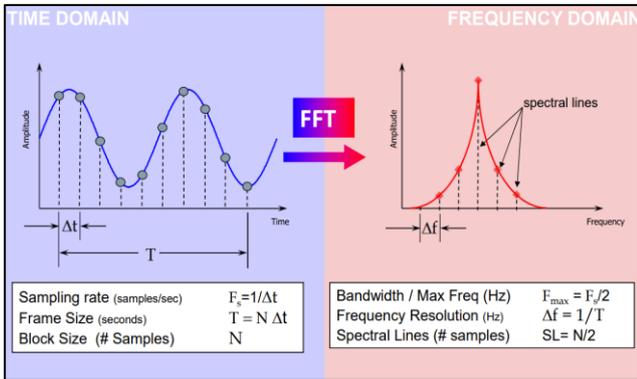
**Figure 10: Data Cleaning**

Above figures 10 and 11 show the functions of various Glyphs that are used in Fatigue design life software during the process of cleaning the raw data. Raw data consists of shock and noises which must be treated in siemens test lab tool that is nothing but data cleaning process. Following are the reasons for noise in data: -



**Figure 11: Conversion of Time Domain to Frequency Domain Data**

Figure 10 and figure 11 shows various processes followed during conversion of time domain data into frequency domain data. The data is given as input to TS input and it flows through the various blocks and shown in figure 9 to get final output from data values display. Figure 12 shows before and after of raw data converted into cleaned data after removal of noise and disturbances.



**Figure 12: Fast Fourier Transform (FFT)**

Above figure 12 explains an overview of how parameters change when the data is converted from time domain into frequency domain with the help of Fast Fourier Transform (FFT).

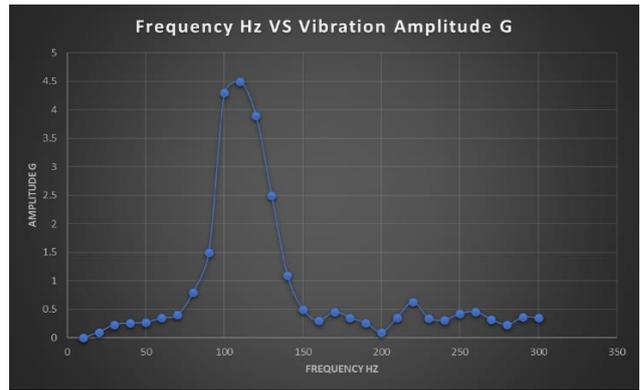
**The sampling rate (Fs)** is important for determining the maximum amplitude and correct waveform of the signal.

The inverse of sampling frequency (Fs) is the sampling interval or  $\Delta t$ . It is the amount of time between data samples collected in the time domain. The smaller the quantity  $\Delta t$ , the better the chance of measuring the true peak in the time domain.

**The block size (N)** is the total number of time data points that are captured to perform a Fourier transform. A block size of 2000 means that two thousand data points are acquired, then a Fourier transform is performed.

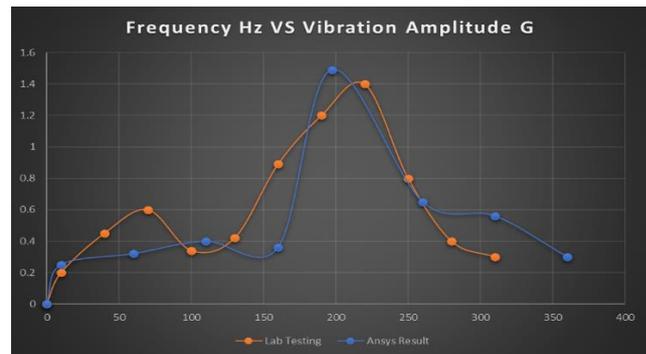
**The frame size** is the total time (T) to acquire one block of data. The total time frame size is also equal to the block size times the time resolution or the frame size is the block size divided by sample frequency.

**VI. RESULTS AND DISCUSSION**



**Figure 13: Frequency (Hz) VS Vibration Amplitude (G)**

- Above figure shows the vibration amplitudes at the input location that is A1 accelerometer. This frequency Vs Vibration amplitude was used as an input to harmonic analysis in Ansys. A peak vibration amplitude of 4.5 was seen at 112 Hz.
- Figure 14 represents the output measured at the response location in actual lab testing and analysis result overlapped on same axis.
- At first, results were plotted in X, Y and Z directions separately and then all the directions were combined to give the worst-case results.
- The curve representing the worst-case scenario i.e., combination of all directions was used for correlation in experimental and analytical validation for correlation activity.
- A1 accelerometer was used for input loading profile and A2 accelerometer was used for response vibration which was used for fatigue damage calculations and design margin along with test analysis correlation activity.



**Figure 14: Frequency (Hz) VS Vibration Amplitude (G)**

**Table 2: Test Analysis Correlation Percentage Difference**

	Frequency (Hz)	Amplitude (G)	GRMS
Ansys Result	197	1.49	13.57
Lab Testing	218	1.4	14.26
% Difference	9.6	6	4.83

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Above table 4 represents Ansys result Vs lab testing in terms of frequency, amplitude and GRMS comparison. Figure 33 shows testing and analysis results mapped on the same graph for comparison. Frequency obtained from test lab matched analysis natural frequency correlation within the acceptable range i.e. under 20%. The high end of SSFR (Structurally significant frequency range) is: High end of SSFR is 150 Hz, for safer side multiplying by 2 that is 300 Hz. Low end of SSFR will be given as: 0 Hz

Modes occurred below this frequency are at risk of being excited by the engine through engine speed. Thus von-mises stress calculations have been done for modes lying in SSFR by scaling to 15g's peak response as experiment measured. Different mode shapes and corresponding natural frequencies at different welding locations with their displacements found in the Ansys results.

Weld 1 experienced 8.8 MPa von mises stress.

Design margin for the same was 2.1. Design margin more than 1 was acceptable.

Remaining 6 welding had experienced stress less than 5 MPa. Where minimum design margin was 3.7 which is acceptable.

## VII. CONCLUSION

- I. Modal scaling and harmonic analysis relation was displayed with simple case study representing analytical and numerical result comparison.
- II. Frequency obtained from test lab matched analysis natural frequency correlation within the acceptable range that is less than 20%.
- III. Close couple AT system passed in both testing and analysis, no failure observed.
- IV. Analysis showed higher threshold G level as compared to testing.
- V. Harmonic profile was generated for close couple AT system and the detailed step by step procedure was documented.

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Ethical Approval and Consent to Participate	No, the article does not require ethical approval and consent to participate with evidence.
Availability of Data and Material/ Data Access Statement	Not relevant.
Authors Contributions	All authors have equal participation in this article.

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